User Note for the Variational EOS Table

H. Togashi[1,2], K. Nakazato[3], H. Suzuki[4], and M. Takano[2]

[1]RIKEN Nishina Center for Accelerator-Based Science, RIKEN, Saitama 351-0198, Japan

[2]Research Institute for Science and Engineering, Waseda University, Tokyo 169-8555, Japan

[3]Faculty of Arts and Science, Kyushu University, Fukuoka 891-0395, Japan

[4]Faculty of Science and Technology, Tokyo University of Science, Chiba 278-8510, Japan

March 8, 2017

Abstract

This is a guide for users of the nuclear equation of state (EOS) table based on the Argonne v18 two-body and Urbana IX three-body potentials. We construct the nuclear EOS using the cluster variational method for uniform matter and the Thomas-Fermi calculation for non-uniform matter.

1 Introduction

We have constructed a nuclear equation of state (EOS) based on the variational many-body theory with realistic nuclear forces, for use in studies such as core-collapse supernovae, black hall formations, and neutron star mergers. More specifically, the EOS of uniform nuclear matter has been calculated with the cluster variational method starting from the Argonne v18 two-body nuclear potential and the Urbana IX three-body nuclear potential. For non-uniform nuclear matter, we have adopted the Thomas-Fermi approximation to obtain the EOS of inhomogeneous phase in a self-consistent manner. Then, we have constructed EOS tables covering wide ranges of density, temperature, and proton fraction. Details of the construction of the EOS are reported in our original paper [1]. This EOS table is open for general use in any studies for nuclear physics and astrophysics. **Please refer to our original paper [1] in your publications when you use our EOS table.**

2 Nuclear EOS table

The EOS table is available on the Web at http://www.np.phys.waseda.ac.jp/EOS/. For convenience, the ranges and grids of the temperature T, proton fraction Y_p, and baryon mass density ρ_B are chosen to be the same as those of the improved Shen EOS in 2011 [2]. The EOS is comprised of the following three tables as in the case of the Shen EOS;

(1) eos.tab (main EOS table)

- Temperature T [MeV]: $-1.00 \leq \log_{10}(T) \leq 2.60$, $\Delta \log_{10}(T) = 0.04$.
- Proton fraction Y_p: $0.01 \leq Y_\mathrm{p} \leq 0.65$, $\Delta Y_\mathrm{p} = 0.01$.
- Baryon mass density ρ_B [g cm^{-3}]: $5.1 \leq \log_{10}(\rho_\mathrm{B}) \leq 16.0$, $\Delta \log_{10}(\rho_\mathrm{B}) = 0.1$.

(2) eos.t00 (EOS table at $T = 0$ MeV)

- Temperature T [MeV]: $T = 0$ MeV.
- Proton fraction Y_p: $0.01 \leq Y_\mathrm{p} \leq 0.65$, $\Delta Y_\mathrm{p} = 0.01$.
- Baryon mass density ρ_B [g cm^{-3}]: $5.1 \leq \log_{10}(\rho_\mathrm{B}) \leq 16.0$, $\Delta \log_{10}(\rho_\mathrm{B}) = 0.1$.

(3) eos.yp0 (EOS table for pure neutron matter at $Y_\mathrm{p} = 0$)

- Temperature T [MeV]: $T = 0$ MeV and $-1.00 \leq \log_{10}(T) \leq 2.60$, $\Delta \log_{10}(T) = 0.04$.
- Proton fraction Y_p: $Y_\mathrm{p} = 0$.
- Baryon mass density ρ_B [g cm^{-3}]: $5.1 \leq \log_{10}(\rho_\mathrm{B}) \leq 16.0$, $\Delta \log_{10}(\rho_\mathrm{B}) = 0.1$.

In the EOS tables, we first fix T which is shown at the beginning of each block in the tables, then fix Y_p to show the thermodynamic quantities for each ρ_B. The blocks with different T are partitioned by a line of characters 'cccccccccccc' as in the case of the Shen EOS table.

The physical quantities listed in the EOS tables are also consistent with those in the Shen EOS except for the effective mass, and are aligned in the following order:

1. Logarithm of baryon mass density: $\log_{10}(\rho_\mathrm{B})[\mathrm{g\ cm}^{-3}]$

2. Baryon number density: $n_\mathrm{B}[\mathrm{fm}^{-3}]$

   The baryon number density n_B of uniform matter is defined by

$$n_\mathrm{B} = n_\mathrm{p} + n_\mathrm{n} + 4n_\alpha, \tag{1}$$

   where n_p, n_n, and n_α are the number densities of protons, neutrons, and alpha particles, respectively.

   For non-uniform matter, n_B is the average nucleon number density defined by

$$n_\mathrm{B} = \frac{4\pi}{V_\mathrm{cell}} \int_0^{R_\mathrm{cell}} \left[n_\mathrm{p}(r) + n_\mathrm{n}(r) + 4n_\alpha(r)\right] r^2 dr, \tag{2}$$

   where $n_\mathrm{p}(r)$, $n_\mathrm{n}(r)$, and $n_\alpha(r)$ are the number density distributions for protons, neutrons, and alpha particles, respectively, in a Wigner-Seitz cell whose volume is $V_\mathrm{cell} = 4\pi R_\mathrm{cell}^3/3$.

   In the EOS tables, the baryon mass density corresponds to the baryon number density as $\rho_\mathrm{B} = m_\mathrm{u} n_\mathrm{B}$ with $m_\mathrm{u}c^2 = 931.494$ MeV.

3. Proton fraction: Y_p

   The proton fraction Y_p of uniform matter is defined by

$$Y_\mathrm{p} = \frac{n_\mathrm{p} + 2n_\alpha}{n_\mathrm{B}}. \tag{3}$$

   For non-uniform matter, Y_p is the average proton fraction defined by

$$Y_\mathrm{p} = \frac{4\pi}{n_\mathrm{B} V_\mathrm{cell}} \int_0^{R_\mathrm{cell}} \left[n_\mathrm{p}(r) + 2n_\alpha(r)\right] r^2 dr. \tag{4}$$

4. Free energy per nucleon: F [MeV]

   The free energy per nucleon F is measured from the nucleon rest mass energy, i.e., F does not include the nucleon rest mass energy. This definition is consistent with that in the Shen EOS. The free energy per nucleon including the rest mass energy is given by

$$\mathcal{F} = F + Y_{\rm p} m_{\rm p} c^2 + (1 - Y_{\rm p}) m_{\rm n} c^2, \tag{5}$$

   with $m_{\rm p}c^2 = 938.272$ MeV and $m_{\rm n}c^2 = 939.565$ MeV.

5. Internal energy per nucleon: $E_{\rm int}$[MeV]

   The internal energy per nucleon $E_{\rm int}$ is measured from $m_{\rm u}c^2$, and is defined by

$$E_{\rm int}(n_{\rm B}, Y_{\rm p}, T) = F + TS + Y_{\rm p} m_{\rm p} c^2 + (1 - Y_{\rm p}) m_{\rm n} c^2 - m_{\rm u} c^2. \tag{6}$$

   Here S is the entropy per nucleon defined bellow.

6. Entropy per nucleon: S [$k_{\rm B}$]

   The entropy per nucleon S is defined by

$$S(n_{\rm B}, Y_{\rm p}, T) = -\left[\frac{\partial F(n_{\rm B}, Y_{\rm p}, T)}{\partial T}\right]_{n_{\rm B}, Y_{\rm p}}. \tag{7}$$

7. Mass number of the heavy nucleus: A

   The mass number of the heavy nucleus A is defined by

$$A = 4\pi \int_0^{R_{\rm A}} \left[n_{\rm p}(r) + n_{\rm n}(r)\right] r^2 dr, \tag{8}$$

   where $R_{\rm A}$ is the "radius" of the heavy nucleus which is defined in the Thomas-Fermi approximation.

8. Proton number of the heavy nucleus: Z

   The proton number of the heavy nucleus Z is defined by

$$Z = 4\pi \int_0^{R_{\rm A}} n_{\rm p}(r)\, r^2 dr. \tag{9}$$

9. Free neutron fraction: $X_{\rm n}$

   The free neutron fraction $X_{\rm n}$ of uniform matter is defined by

$$X_{\rm n} = \frac{n_{\rm n}}{n_{\rm B}}. \tag{10}$$

   For non-uniform matter, $X_{\rm n}$ is defined by

$$X_{\rm n} = \frac{4\pi}{n_{\rm B} V_{\rm cell}} \int_{R_{\rm A}}^{R_{\rm cell}} n_{\rm n}(r)\, r^2 dr. \tag{11}$$

10. Free proton fraction: $X_{\rm p}$

   The free proton fraction $X_{\rm p}$ of uniform matter is defined by

$$X_{\rm p} = \frac{n_{\rm p}}{n_{\rm B}}. \tag{12}$$

   For non-uniform matter, $X_{\rm p}$ is defined by

$$X_{\rm p} = \frac{4\pi}{n_{\rm B} V_{\rm cell}} \int_{R_{\rm A}}^{R_{\rm cell}} n_{\rm p}(r)\, r^2 dr. \tag{13}$$

11. Free alpha-particle fraction: X_α

   The free alpha-particle fraction X_α of uniform matter is defined by

$$X_\alpha = \frac{4 n_\alpha}{n_{\rm B}}. \tag{14}$$

   For non-uniform matter, X_α is defined by

$$X_\alpha = \frac{16\pi}{n_{\rm B} V_{\rm cell}} \int_{0}^{R_{\rm cell}} n_\alpha(r)\, r^2 dr. \tag{15}$$

12. Heavy nucleus fraction: $X_{\rm A}$

   The heavy nucleus fraction $X_{\rm A}$ is defined by

$$X_{\rm A} = \frac{A}{n_{\rm B} V_{\rm cell}}. \tag{16}$$

13. Pressure: P [MeV fm^{-3}]

    The pressure P is defined by

$$P(n_{\rm B}, Y_{\rm p}, T) = n_{\rm B}^2 \left[\frac{\partial F(n_{\rm B}, Y_{\rm p}, T)}{\partial n_{\rm B}} \right]_{Y_{\rm p}, T}. \tag{17}$$

14. Neutron chemical potential: $\mu_{\rm n}$ [MeV]

    The neutron chemical potential $\mu_{\rm n}$ is defined by

$$\mu_{\rm n}(n_{\rm B}, Y_{\rm p}, T) = \left[\frac{\partial (n_{\rm B} F(n_{\rm B}, Y_{\rm p}, T))}{\partial \bar{n}_{\rm n}} \right]_{\bar{n}_{\rm p}, T}. \tag{18}$$

    Here $\bar{n}_{\rm p} = Y_{\rm p} n_{\rm B}$ and $\bar{n}_{\rm n} = (1 - Y_{\rm p}) n_{\rm B}$ are the average number densities of protons and neutrons, respectively. In the table, $\mu_{\rm n}$ is measured from the neutron rest mass energy $m_{\rm n} c^2$.

15. Proton chemical potential: $\mu_{\rm p}$ [MeV]

    The proton chemical potential $\mu_{\rm p}$ is defined by

$$\mu_{\rm p}(n_{\rm B}, Y_{\rm p}, T) = \left[\frac{\partial (n_{\rm B} F(n_{\rm B}, Y_{\rm p}, T))}{\partial \bar{n}_{\rm p}} \right]_{\bar{n}_{\rm n}, T}. \tag{19}$$

    In the table, $\mu_{\rm p}$ is measured from the proton rest mass energy $m_{\rm p} c^2$.

A similar list of the thermodynamic quantities in the EOS table is also given in the Appendix of Ref. [1], wherein some of the definitions and explicit expressions for the above-mentioned physical quantities are explained in more detail.

3 Contact

If you find any problems, please contact us. We would appreciate it very much if you could give us comments or suggestions on this EOS table. Please contact the following author;

- Hajime Togashi

  RIKEN Nishina Center for Accelerator-Based Science, RIKEN,

  2-1 Hirosawa, Wako, Saitama 351-0198, Japan

  E-mail: hajime.togashi@riken.jp

References

[1] H. Togashi, K. Nakazato, Y. Takehara, S. Yamamuro, H. Suzuki and M. Takano, Nucl. Phys. A 961 (2017) 78.

[2] H. Shen, H. Toki, K. Oyamatsu and K. Sumiyoshi, Astrophys. J. Suppl. 197 (2011) 20.